Combinatorial Optimization Research

Applications in Industry and Transportation

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Overview of problems discussed

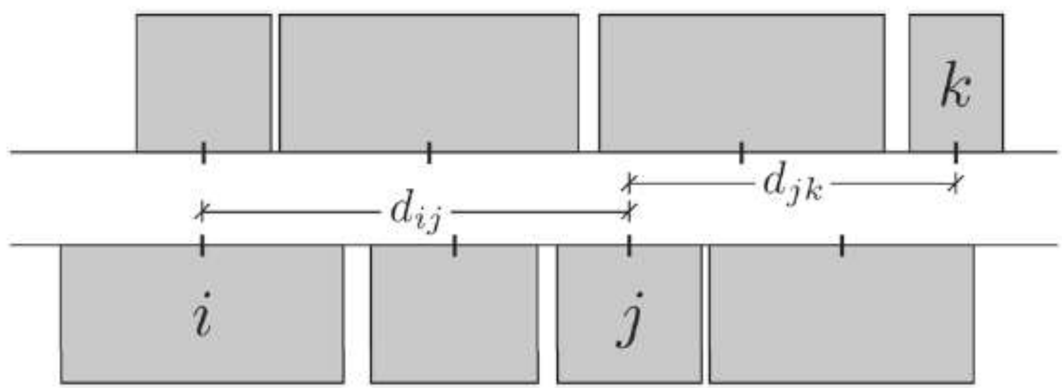
- Double-Row Layout Problem (DRLP)
- Transportation of Wheelchair Users
- Nesting Problem in Ship Cargo Optimization
- Aircraft Refueling Problem at Airports

Double-Row Layout Problem (DRLP)

- Arranging machines along rows in factories

n	Number of machines
$N = \{1, \ldots, n\}$	Set of machines
$R =$ {upper row, lower row}	Set of rows that define a corridor
c_{ij}	Amount of flow between machines i e j $(1 \leq i < j \leq n)$
ℓ_i	Length of machine i
$L = \sum_{i=1}^{n} \ell_i$	

Fig. 1 A double row layout. The distance d_{ij} between two machines i and j is the horizontal distance between their centers



DRLP

- **Objective**: Minimize material handling costs
- **Solution Methods**: Heuristics, mathematical models

A mathematical formulation for the DRLP is given by:

$$\min_{\varphi \in \Phi_n} \sum_{1 \leq i < j \leq n} c_{ij} d_{ij}^{\varphi} \qquad (1)$$

where Φ_n is the set of all double row layouts on the set N; and d_{ij}^{φ} is the distance between machines i and j with reference to a layout $\varphi \in \Phi_n$.

DRLP

A mixed-integer programming model

- Define the variables:

x_i	Abscissa of the center of machine i $(1 \leq i \leq n)$
d_{ij}	Distance between machines i e j $(1 \leq i < j \leq n)$
α_{ij}	Binary variable that equals 1 if machine i is on the left of machine j, both on the same row; and 0 otherwise $(1 \leq i,\ j \leq n, i \neq j)$

DRLP

- Consider the vector:

$$\alpha = (\alpha_{ij})_{1 \le i, j \le n; i \ne j} \in \{0, 1\}^{n(n-1)}$$

- α is the incidence vector of a partition of the set of *n* machines into two linear orders.
- the convex hull of all α-incidence vectors forms a polytope, denoted as P_n.
- Find classes of inequalities that are valid for the convex hull of points defined by P_n.

Proposition 1 *The following inequalities are valid for* P_n*:*

$$-\alpha_{ij} + \alpha_{ik} + \alpha_{jk} - \alpha_{ji} + \alpha_{ki} + \alpha_{kj} \leq 1, \quad (i, j, k \in N; i < j; k \neq i, k \neq j) \tag{2}$$

$$-\alpha_{ij} + \alpha_{ik} - \alpha_{jk} + \alpha_{ji} - \alpha_{ki} + \alpha_{kj} \leq 1, \quad (i, j, k \in N; i < j; k < j; i \neq k) \tag{3}$$

Proposition 2 *The following inequality is valid for* P_n*:*

$$\alpha_{ij} + \alpha_{ik} + \alpha_{jk} + \alpha_{ji} + \alpha_{ki} + \alpha_{kj} \geq 1, \quad (1 \leq i < j < k \leq n) \tag{4}$$

The following trivial inequalities are also valid for P_n:

$$0 \leq \alpha_{ij}, \alpha_{ij} \leq 1, \quad (1 \leq i, j \leq n; i \neq j) \tag{5}$$

DRLP

In the sequel we shall consider the polytope $Q_n(\supseteq P_n)$ defined by:

$$Q_n = \{\alpha \in R^{n(n-1)} : (2), (3), (4), \text{ and } (5)\}.$$

Remark 1 The integral points in Q_n are precisely the α-incidence vectors, which means that $P_n \equiv conv(\alpha \in Q_n : \alpha \text{ is integral})$.

MIP model of the DRLP- Amaral (2013)

$$\min \sum_{(i,j);i<j} c_{ij} d_{ij} \qquad (1)$$

$$d_{ij} \geq x_i - x_j, \qquad i < j \qquad (2)$$

$$d_{ij} \geq x_j - x_i, \qquad i < j \qquad (3)$$

$$d_{ij} - \alpha_{ij}(\ell_i + \ell_j)/2 - \alpha_{ji}(\ell_i + \ell_j)/2 \geq 0, \qquad i < j \qquad (4)$$

$$x_i + (\ell_i + \ell_j)/2 \leq x_j + L(1 - \alpha_{ij}), \qquad i \neq j \qquad (5)$$

$$x_p \leq x_q, \qquad (p, q) = \arg\min_{i<j} c_{ij} \qquad (6)$$

$$-\alpha_{ij} - \alpha_{ji} + \alpha_{ik} + \alpha_{ki} + \alpha_{jk} + \alpha_{kj} \leq 1, \qquad i < j,\ k \neq i,\ k \neq j \qquad (7)$$

$$-\alpha_{ij} + \alpha_{ji} + \alpha_{ik} - \alpha_{ki} - \alpha_{jk} + \alpha_{kj} \leq 1, \qquad i < j,\ k < j,\ k \neq i \qquad (8)$$

$$\alpha_{ij} + \alpha_{ji} + \alpha_{ik} + \alpha_{ki} + \alpha_{jk} + \alpha_{kj} \geq 1, \qquad i < j < k \qquad (9)$$

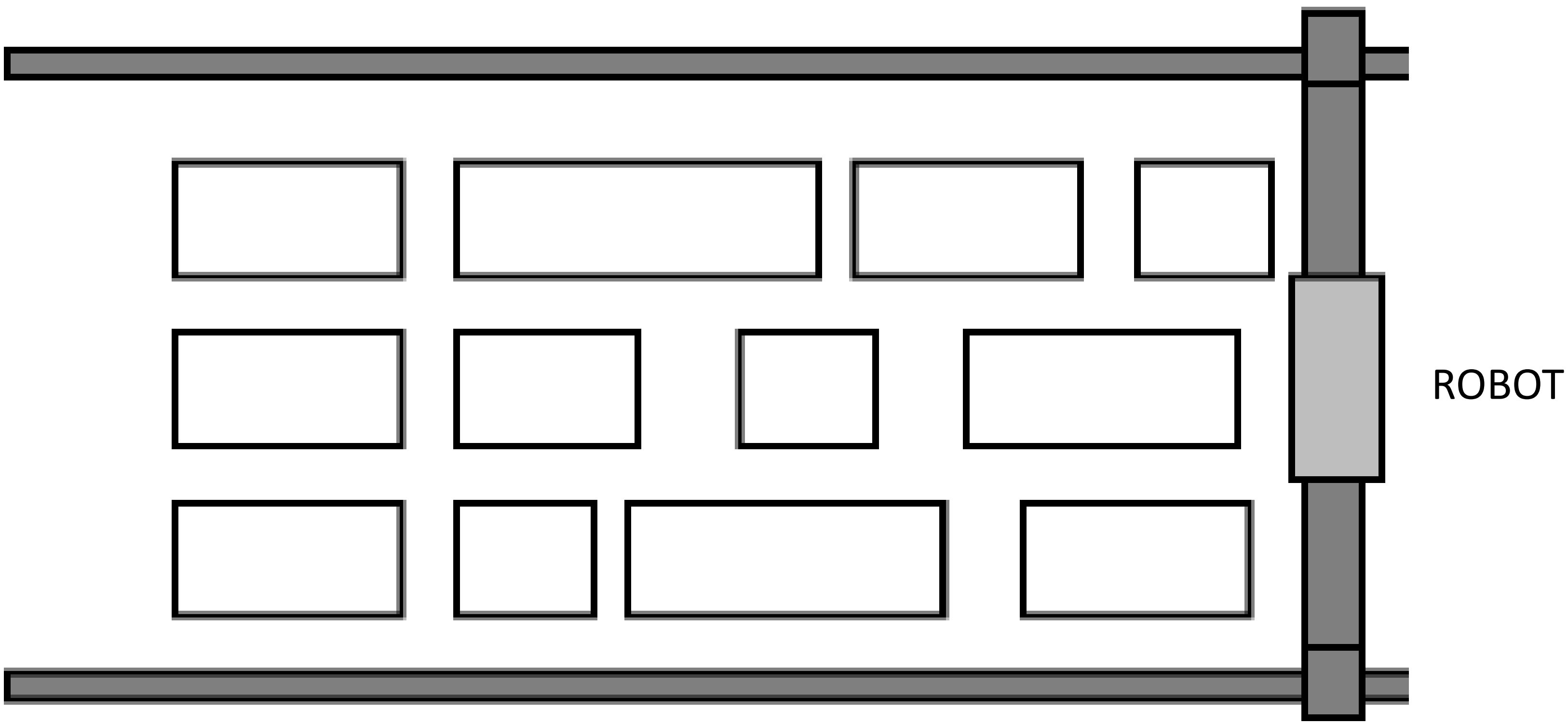
$$\alpha_{ij} \in \{0, 1\}, \qquad i \neq j \qquad (10)$$

$$x_i \in [\ell_i/2,\ L - \ell_i/2], \qquad 1 \leq i \leq n \qquad (11)$$

Research Progress on the Double Row Layout Problem

- 2010: Chung and Tanchoco proposed a MIP model and obtained exact solutions for problems with **$n \le 10$**.
- More efficient models for double row layouts were proposed by Amaral (2013) and Amaral (2018), solving problems with **$n \le 13$**.
- Secchin and Amaral (2018) proposed a model solving problems with **$n \le 15$**.
- Chae and Regan (2020) modified the Secchin-Amaral model and solved problems with **$n \le 16$**.
- Amaral (2021) proposed a mathematical model solving problems with **$n \le 16$**, more efficiently than the Chae-Regan model.
- Recently, Amaral(2024) gave a MIP that solved problems with **$n \le 20$**.

Future research:
Multi-Row Layout Problem



Transportation of Wheelchair Users Dial-a-Ride Problem (DARP)

- Optimizing routes for adapted buses

Vitória-ES

Serviço Mão-na-Roda
Metropolitan Area of Vitória-Brazil

Summary - Dial-a-Ride Problem (DARP)

- DARP is a combinatorial NP-hard problem, computationally complex to solve.
- It is a variant of the Vehicle Routing Problem (VRP), but it specifically focuses on passenger transportation.
- **INPUT**: pickup and delivery requests and a fleet of vehicles.
- **GOAL**: generate optimal routes that ensure all requests are serviced while minimizing costs.

DARP

The key constraints include:

- **Vehicle capacity**: Each vehicle can only carry a limited number of passengers.
- **Maximum ride time**: Passengers must not spend too much time on the vehicle.
- **Time windows**: Pickup and drop-off must happen within specific time intervals.

Additional Constraints

- The **duration of the route taken by vehicle *k*** must not too long.
- The **waiting time at any location** must not exceed the allowed maximum.

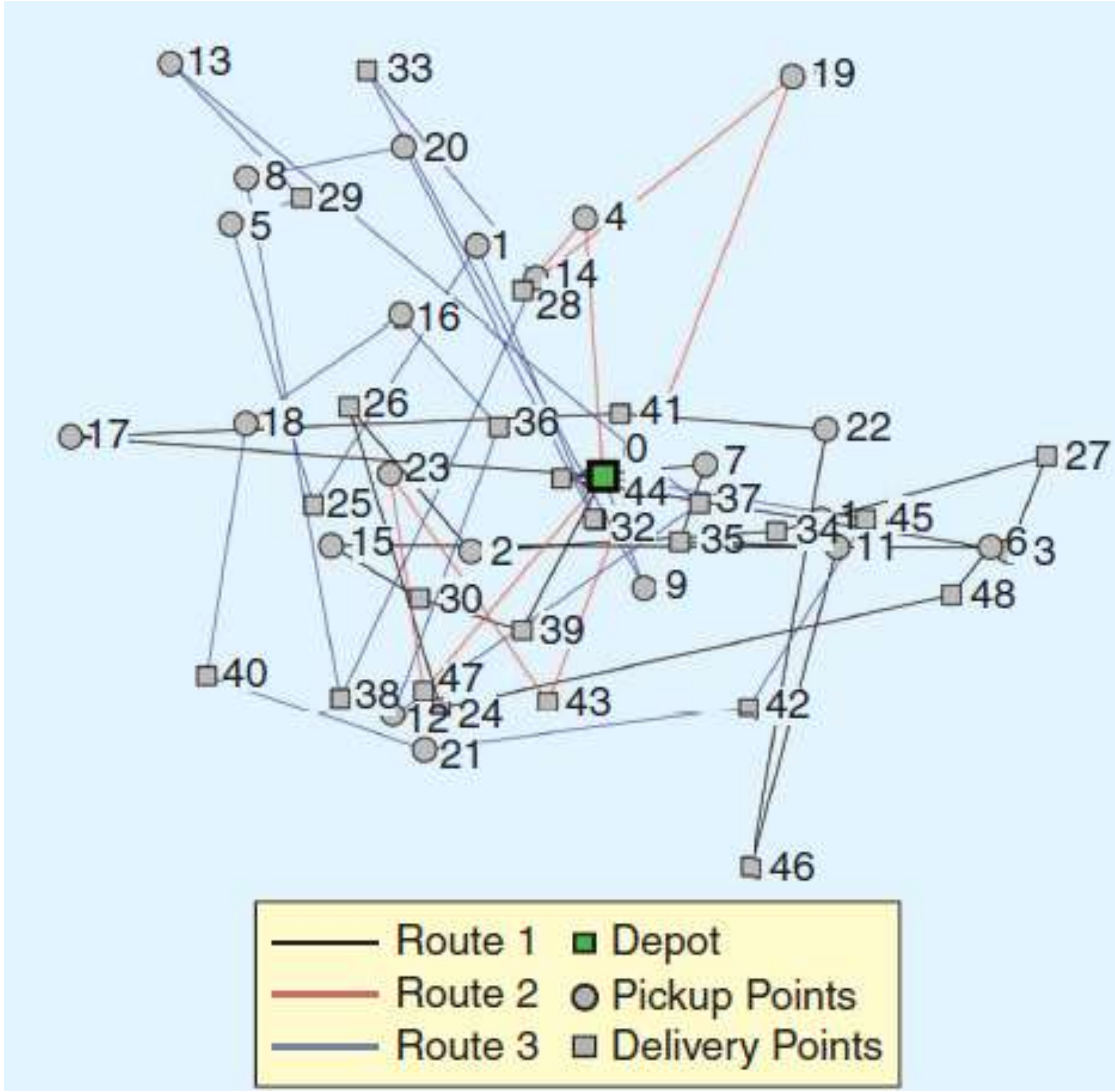


Figure in: Mauri-Lorena(2009), IEEE INTELLIGENT TRANSPORTATION SYSTEMS MAGAZINE 6 10.1109/MITS.2009.934641

DARP/Publication

Campos, Alba Assis ; Amaral, André Renato Sales . An Iterated Local Search for the Multi-objective Dial-a-Ride Problem. Advances in Intelligent Systems and Computing. 1ed.: Springer International Publishing, 2021, v. 1351, p. 1302-1313.

Nesting Problem in Ship Cargo Optimization

- Fitting rectangular steel plates in ships
- **Objective**: Maximize space usage, minimize waste
- **Application**: Maritime shipping

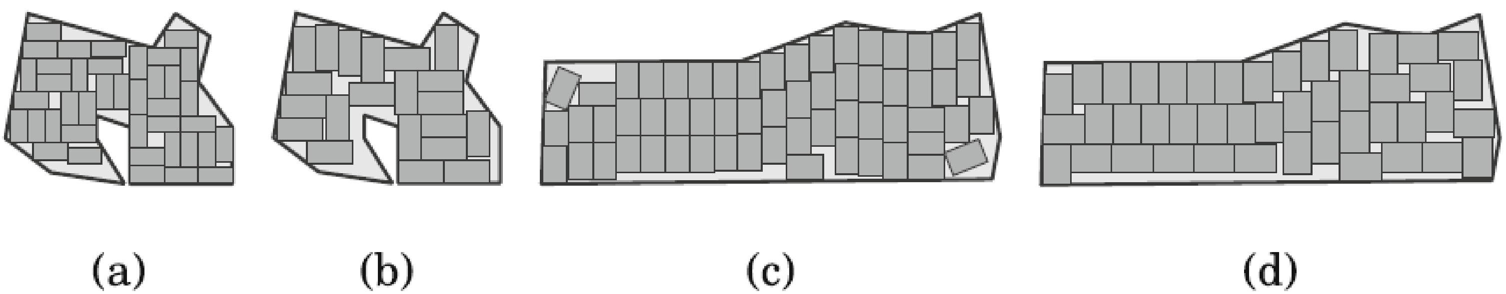


Fig. 6. Best layouts found: (a) 40 items in artif-01-c; (b) 21 in artif-02-c; (c) 60 in artif-03-a; and (d) 45 in artif-04-c.

Nesting/Publication

Romanelli, Alexandre ; Amaral, André R. S. . A Solution Approach to The Problem of Nesting Rectangles with Arbitrary Rotations into Containers of Irregular Convex and Non-Convex Shapes. In: Lalla-Ruiz E.; Mes M.; Voss S.. (Org.). Lecture Notes in Computer Science. Cham, Switzerland: Springer International Publishing, 2020, v. 12433, p. 747-762.

Aircraft Refueling Problem

- Routing fuel trucks at airports
- **Constraints**: Truck availability, time windows
- **Application**: Smooth airport operations
- **Methods**: Metaheuristics, MIP


air bp


A02
Brasil
CTA
1525
CTA
1523
BR

Table 1. List of parameters for each node.

Par	Description	*Par*	Description
$D_{i,j}$	Distance between nodes $i, j \in N$	c_i	Fuel quantity requested by aircraft i
a_i	Earliest moment at which refueling can start	b_i	Latest possible starting time for refueling
d_i	Distance between parked aircraft i and the storage area	q_i	Refueling rate for aircraft i

Table 2. List of parameters for each truck $k \in K$.

Par	Description
V_k	Average velocity of truck k
Q_k	Maximum fuel capacity of truck k

Aircraft Refueling/ Publication

- Zampirolli, Karyne Alves ; Amaral, André Renato Sales. Simulated Annealing and Iterated Local Search Approaches to the Aircraft Refueling Problem. Lecture Notes in Computer Science. 1ed.Cham: Springer International Publishing, 2021, v. 12952, p. 422-438.

Questions and Discussion

Thank You!!