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SEMINÁRIO DE ANÁLISE E EQUAÇÕES DIFERENCIAIS

Dia 4 de Janeiro (quinta-feira), às 13H30, na sala 6.2.33

Stability for the Data Processing Inequality in Quantum Information Theory

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abstract The Data Processing Inequality (DPI) says that the quantum relative entropy $S(\rho||\sigma) := \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$ is non-increasing under the action of completely positive trace preserving (CPTP) maps. Let $\mathcal{M}$ be a finite dimensional von Neumann algebra and $\mathcal{N}$ a von Neumann subalgebra if it. Let $\mathcal{E}_\tau$ be the tracial conditional expectation from $\mathcal{M}$ onto $\mathcal{N}$. For density matrices ρ and σ in $\mathcal{M}$, let $\rho_\mathcal{N} := \mathcal{E}_\tau\rho$ and $\sigma_\mathcal{N} := \mathcal{E}_\tau\sigma$. Since $\mathcal{E}_\tau$ is CPTP, the DPI says that $S(\rho||\sigma) \geq S(\rho_\mathcal{N}||\sigma_\mathcal{N})$, and the general case is readily deduced from this. A theorem of Petz says that there is equality if and only if $\sigma = \mathcal{R}_\rho(\sigma_\mathcal{N})$, where $\mathcal{R}_\rho$ is the *Petz recovery map* whose origin and properties will be explained. In it simplest form, our bound is

$$S(\rho||\sigma) - S(\rho_\mathcal{N}||\sigma_\mathcal{N}) \geq \left(\frac{1}{8\pi}\right)^4 \|\Delta_{\sigma,\rho}\|^{-2}\|\mathcal{R}_\rho(\sigma_\mathcal{N}) - \sigma\|_1^4 \ .$$

where $\Delta_{\sigma,\rho}$ is the relative modular operator. We also prove related results for various quasi-relative entropies. The talk will require no knowledge of von Neumann algebras; the results will be presented in the finite dimensional case where they are already interesting both as mathematics and from the point of view of quantum information theory. This is joint work with Anna Veshysnina.

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